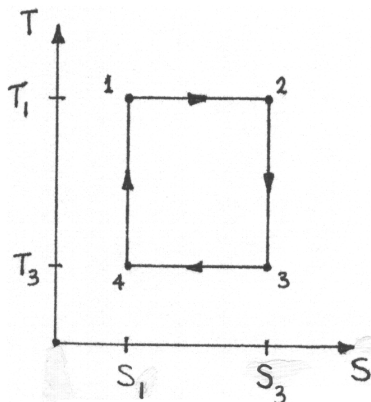


# EXAMINATION 1

**Directions.** Do all four problems (weights are indicated). This is a closed-book closed-note exam except for one  $8\frac{1}{2} \times 11$  inch sheet containing any information you wish on both sides. You are free to approach the proctor to ask questions – but he or she will not give hints and will be obliged to write your question and its answer on the board. Don't use a calculator, which you don't need – roots, circular functions, *etc.*, may be left unevaluated if you do not know them. Use a bluebook. Do not use scratch paper – otherwise you risk losing part credit. Cross out rather than erase any work that you wish the grader to ignore. Justify what you do. Box or circle your answer.

1. (25 points) A heat engine for which the working material is an ideal monatomic gas moves slowly enough that all parts of it are always in mutual equilibrium. It is described by a rectangular path on the  $T$  (absolute temperature) –  $S$  (entropy) plane, as in the figure. While on the path  $1 \rightarrow 2$ , the gas in the engine takes heat from a bath at high temperature  $T_1$ ; on the path  $3 \rightarrow 4$ , it returns heat to bath at lower temperature  $T_3$ . On the paths  $2 \rightarrow 3$  and  $4 \rightarrow 1$ , the entropy has constant values  $S_3$  and  $S_1$ , respectively.



a. (5 points) Write down the net change

$$(\Delta U_{23} + \Delta U_{41})$$

in internal energy for the sum of the two paths  $2 \rightarrow 3$  and  $4 \rightarrow 1$ .

b. (5 points) Compute the net change

$$\oint_{12341} T dS$$

over one complete cycle of the engine.

c. (8 points) Deduce the value of the mechanical work

$$\oint_{12341} p dV$$

done **by** the gas **on** the rest of the universe over one complete cycle of the engine.

d. (7 points) In one cycle, what fraction of the heat withdrawn from the hot reservoir is converted to mechanical work done by the gas on the rest of the universe?

*Hint: Keep in mind that the only parameters given in this problem are  $T_1$ ,  $T_3$ ,  $S_1$ , and  $S_3$ ; your answers, if nontrivial, must be expressed in terms of these parameters.*

2. (25 points) In a hypothetical one-dimensional system, thermal motion of atoms in the  $y$  and  $z$  directions is “frozen out”, so, effectively, the atoms are able to move only in the  $x$  direction. In that direction, an atom has velocity  $v$  ( $-\infty < v < \infty$ ). The fraction  $dF$  of atoms with velocity between  $v$  and  $v + dv$  is

$$dF \equiv f_v(v) dv = \frac{\exp\left(-\frac{mv^2}{2kT}\right) dv}{\int_{-\infty}^{\infty} \exp\left(-\frac{mv^2}{2kT}\right) dv},$$

where  $f_v(v)$  is the probability density (RHK: “relative probability”) of the value  $v$ ,  $m$  is the atomic mass,  $k$  is Boltzmann’s constant, and  $T$  is the absolute temperature.

a. (10 points) Calculate the mean value of the square of  $v$ , *i.e.*  $\langle v^2 \rangle$ . If you wish, you may

leave your answer in the form of a ratio of definite integrals. *Do not merely guess the answer.*

- b. (15 points) Define  $E$  to be the kinetic energy  $\frac{1}{2}mv^2$  of an atom. The fraction  $dF$  of atoms with kinetic energy between  $E$  and  $E+dE$  is

$$dF \equiv f_E(E) dE ,$$

where  $f_E(E)$  is the probability density of the value  $E$ . One might imagine  $f_E(E)$  to take the possible forms:

$$f_E(E) \propto E^{-1/2} \exp\left(-\frac{E}{kT}\right) ?$$

$$\propto \exp\left(-\frac{E}{kT}\right) ?$$

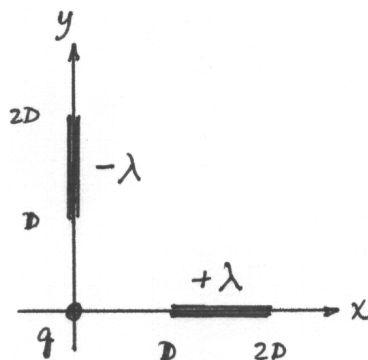
$$\propto E^{1/2} \exp\left(-\frac{E}{kT}\right) ?$$

$$\propto E \exp\left(-\frac{E}{kT}\right) ?$$

Which one form is correct, and why?

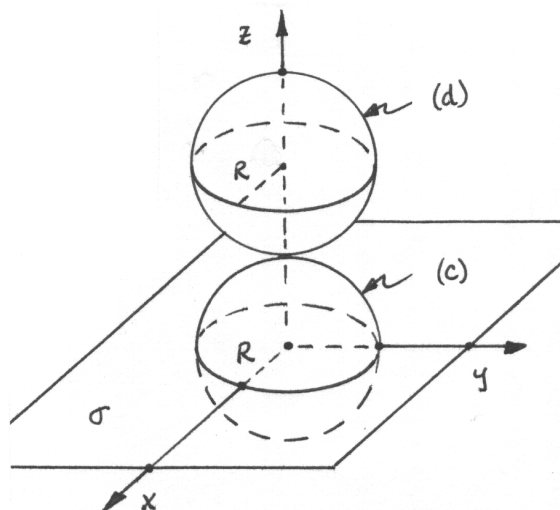
3. (25 points)

A fixed line charge of  $+\lambda$  esu/cm on the  $x$  axis extends from  $x = D$  to  $x = 2D$ , and a fixed line charge of  $-\lambda$  esu/cm on the  $y$  axis extends from  $y = D$  to  $y = 2D$ .



- a. (10 points) Find the work required to bring a test point charge  $q$  from infinity to the origin. Does your answer depend on the path you chose? If so, specify the path.
- b. (15 points) Calculate the mechanical force (magnitude and direction) that is required to keep the test charge at the origin.

4. (25 points) The infinite plane  $z = 0$  carries a uniform surface charge density  $\sigma$  esu/cm<sup>2</sup>. There are no other charges in the problem.



- a. (5 points) Find the magnitude and direction of the electric field  $\mathbf{E}_+$  everywhere in the region  $z > 0$ .
- b. (5 points) Find the magnitude and direction of the electric field  $\mathbf{E}_-$  everywhere in the region  $z < 0$ .
- c. (8 points) Consider a spherical surface of radius  $R$  centered at the origin. Find the electric flux

$$\iint \mathbf{E} \cdot d\mathbf{a}$$

through the *top half* (top hemisphere) of this surface.

- d. (7 points) Consider a second spherical surface, again of radius  $R$ , but now centered at the point  $(0,0,2R)$ , so that it does not enclose any charge. Find the electric flux

$$\iint \mathbf{E} \cdot d\mathbf{a}$$

through the *bottom half* (bottom hemisphere) of this surface.